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A LEAST-SQUARES TECHNIQUE FOR THE LOCATION OF HYDROPHONES BY THE USE OF VANDERKULK SURVEY DATA

By

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15 December 1969

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CONTENTS

SUMMARY

The method devised by W. Vanderkulk for the relative location of three hydrophones in "Remarks on a Hydrophone Location Method," JOURNAL OF UNDERWATER ACOUSTICS, Vol. II, No. 2, April 1961, yields redundant data for the interhydrophone distances when applied to an entire array. The purpose of this report is to develop the mathematics for a least-squares method to use the redundant measurements to yield the coordinates of all the hydrophones within the array at the Pacific Missile Range Facility, Hawaiian Area Underwater Range.

INTRODUCTION

The method developed by W. Vanderkulk* is an ingenious technique for establishing the relative location of three hydrophones, however, problems develop when an attempt is made to extend the method to locate an entire array of hydrophones.

The scheme first employed was to calculate the interhydrophone distances from each set of triads and then to build up the array from some particular starting triad. The obvious problem is that since the results of the Vanderkulk calculations have some error, the common distance between two hydrophones will not be the same when measured in two different triads. Thus there can be no set of hydrophone positions that will exactly fit all the measured distances. Some arbitrary judgment must be made, and the derived hydrophone positions must be constantly revised to compensate for the discrepancy between the calculated and the measured distances.

As would be expected, the errors appear to be more pronounced with increasing distance from the starting triad.

The standard technique in dealing with redundant measurements is that of least squares, it is the purpose of this report to develop a least-squares method of using the Vanderkulk information to simultaneously locate the entire array of hydrophones.

DEVELOPMENT OF THE NORMAL EQUATIONS

The first step in a least-squares procedure is to devise the function to be minimized. Since the most meaningful output from the Vanderkulk calculations is the interhydrophone distances, these distances will be the basis for the minimization. Thus, let

$$S = \sum W_i \left(D_{mi} - D_{ti}\right)^2 \tag{1}$$

where the D_{mi} represents the measured distances, the D_{ti} represents the "true" distance between pairs of hydrophones, and the W_i are the relative weights of the *ith* measurements.

*Department of Defense. "Remarks on a Hydrophone Location Method," by W. Vanderkulk, J. UNDERWATER ACOUSTICS, Vol. II, No. 2 (Apr 1961), pp. 241-250.

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The next step is to derive a linear relation between the hydrophone coordinates and the distance. The following derivation will consider only the X–Y grid coordinates and not the full three-dimensional location problem.

Let the coordinates of the IJ*th* hydrophone (I represents the row, J the column) be X_{ij}, Y_{ij}; then

$$D_{ij,k\ell} = \sqrt{\left(X_{ij} - X_{k\ell}\right)^2 + \left(Y_{ij} - Y_{k\ell}\right)^2}$$

Now, let $X_{ij}^{(N)}$ be an estimate of X_{ij} and $Y_{ij}^{(N)}$ an estimate of Y_{ij}, or

$$X_{ij} = X_{ij}^{(N)} + \Delta_{ijX}$$

$$Y_{ij} = Y_{ij}^{(N)} + \Delta_{ijY}$$

Introducing the notation

$$D_{ij,k\ell}^{(N)} = \sqrt{\left(X_{ij}^{(N)} - X_{k\ell}^{(N)}\right)^2 + \left(Y_{ij}^{(N)} - Y_{k\ell}^{(N)}\right)^2}$$

$$D(\Delta) = \sqrt{\left(\Delta_{ijX} - \Delta_{k\ell X}\right)^2 + \left(\Delta_{ijY} - \Delta_{k\ell Y}\right)^2}$$

$$D^{(N)}(X) = \left(X_{ij}^{(N)} - X_{k\ell}^{(N)}\right)$$

$$D^{(N)}(Y) = \left(Y_{ij}^{(N)} - Y_{k\ell}^{(N)}\right)$$

$$a_{ij,k\ell}^{(N)} = \frac{D^{(N)}(X)}{D_{ij,k\ell}^{(N)}}$$

$$b_{ij,k\ell}^{(N)} = \frac{D^{(N)}(Y)}{D_{ij,k\ell}^{(N)}}$$

we have

$$D_{ij,k\ell} = \left[\left(D_{ij,k\ell}^{(N)}\right)^2 + 2D^{(N)}(X)\left(\Delta_{ijX} - \Delta_{k\ell X}\right) + 2D^{(N)}(Y)\left(\Delta_{ijY} - \Delta_{k\ell Y}\right) + D^2(\Delta)\right]^{\frac{1}{2}}$$

Now, assume that second-order differences are negligible and expand the terms under the radical:

$$D_{ij,k\ell} \simeq D_{ij,k\ell}^{(N)} + a_{ij,k\ell}^{(N)}\left(\Delta_{ijX} - \Delta_{k\ell X}\right) + b_{ij,k\ell}^{(N)}\left(\Delta_{ijY} - \Delta_{k\ell Y}\right) \qquad \text{(II)}$$

Substitution of (II) into (I) yields:

$$S \approx \sum W_{ij,k\ell}\left\{D_{m(ij,k\ell)} - \left[D^{(N)}_{ij,k\ell} + a^{(N)}_{ij,k\ell}\left(\Delta_{ijX} - \Delta_{k\ell X}\right) + b^{(N)}_{ij,k\ell}\left(\Delta_{ijY} - \Delta_{k\ell Y}\right)\right]\right\}^2 \qquad \text{(III)}$$

Thus, taking the partial derivatives of S with respect to the Δ_{ijX} and Δ_{ijY} and equating to zero will yield the standard set of normal equations.

The solution to the normal equations will be the best estimate of the hydrophone X-Y coordinates in the linearized least-squares sense.

DERIVATION OF THE INITIAL ESTIMATE

The system of equations generated by the minimization of equation (III) involves an iterative technique.

The derivation that follows is the development of a good first approximation and the resulting set of equations for the particular configuration at the Pacific Missile Range Facility, Hawaiian Area Underwater Range. This range can nominally be represented as a collection of 52 equilateral triangles with sides of 8,000 feet (see figure 1). The center of the range is hydrophone 3-4, and the positive Y-axis is determined by the line from hydrophone 3-4 to hydrophone 3-7.

The nominal location of the X-Y coordinates for each hydrophone IJ is

$$\left[(2r-1),s\right] = \left[(2-r)\sqrt{3}\,d,(s-4)d\right] \qquad (r = 1,2,3)\ (s = 1,\ldots,7)$$

$$(2r,s) = \left[\frac{3-2r}{2}\,3\sqrt{d},\left(s-\frac{9}{2}\right)d\right] \qquad (r = 1,2) \quad (s = 1,\ldots,8)$$

where

d = 8,000 feet

and

r = row

s = column

Now, the true location of each hydrophone can be represented by this nominal location and some additional displacement:

$$\left[(2r-1),s\right] = \left[(2-r)\sqrt{3}\,d + \epsilon_{(2r-1)sx},(s-4)d + \epsilon_{(2r-1)sy}\right]$$

$$(2r,s) = \left[\frac{3-2r}{2}\sqrt{3}\,d + \epsilon_{2rsx},\left(s-\frac{9}{2}\right)d + \epsilon_{2rsy}\right]$$

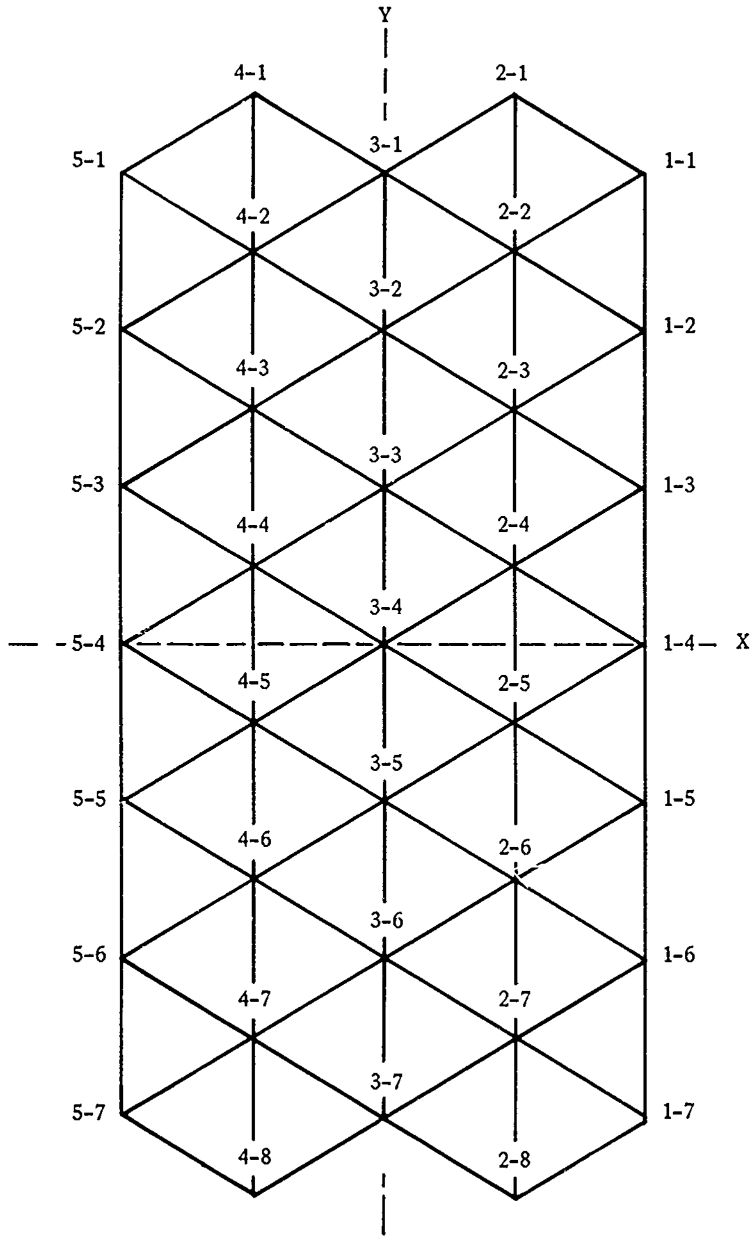

Figure 1. Hydrophone Array, Pacific Missile Range Facility, Hawaiian Area Underwater Range.

In terms of these coordinates, the distance between any two possible adjacent hydrophones is

$$D_{(2r-1)s,(2r-1)(s+1)} = \sqrt{\left(\epsilon_{(2r-1)sx} - \epsilon_{(2r-1)(s+1)x}\right)^2 + \left(-d - \epsilon_{(2r-1)(s+1)y} + \epsilon_{(2r-1)sy}\right)^2}$$

$$D_{(2r-1)s,2rs} = \sqrt{\left(\frac{\sqrt{3}}{2} d - \epsilon_{(2r-1)sx}\right)^2 + \left(\frac{d}{2} - \epsilon_{2rsy} + \epsilon_{(2r-1)sy}\right)^2}$$

$$D_{(2r-1)s,2r(s+1)} = \sqrt{\left(\frac{\sqrt{3}}{2} d - \epsilon_{2r(s+1)x} + \epsilon_{(2r-1)sx}\right)^2 + \left(-\frac{d}{2} - \epsilon_{(2r-1)sy}\right)^2}$$

$$D_{2rs,2r(s+1)} = \sqrt{\left(\epsilon_{2rsx} - \epsilon_{2r(s+1)x}\right)^2 + \left(-d - \epsilon_{2r(s+1)y} + \epsilon_{2rsy}\right)^2}$$

$$D_{2rs,(2r+1)(s-1)} = \sqrt{\left(\frac{\sqrt{3}}{2} d - \epsilon_{(2r+1)(s-1)x} + \epsilon_{2rsx}\right)^2 + \left(\frac{d}{2} - \epsilon_{(2r+1)(s-1)y} + \epsilon_{2rsy}\right)^2}$$

$$D_{2rs,(2r+1)s} = \sqrt{\left(\frac{\sqrt{3}}{2} d - \epsilon_{(2r+1)sx} + \epsilon_{2rsy}\right)^2 + \left(-\frac{d}{2} - \epsilon_{(2r+1)sy} + \epsilon_{2rsy}\right)^2}$$

Next, using a linearized expansion,

$$D_{(2r-1)s,(2r-1)(s+1)} \simeq d - \left(\epsilon_{(2r-1)sy} - \epsilon_{(2r-1)(s+1)y}\right)$$

$$D_{(2r-1)s,2rs} \simeq d + \frac{\sqrt{3}}{2}\left(\epsilon_{(2r-1)sx} - \epsilon_{2rsx}\right) + \frac{1}{2}\left(\epsilon_{(2r-1)sy} - \epsilon_{2rsy}\right)$$

$$D_{(2r-1)s,2r(s+1)} \simeq d + \frac{\sqrt{3}}{2}\left(\epsilon_{(2r-1)sx} - \epsilon_{2r(s+1)y}\right)$$

$$D_{2rs,2r(s+1)} \simeq d - \left(\epsilon_{2rsy} - \epsilon_{2r(s+1)y}\right)$$

$$D_{2rs,(2r+1)(s-1)} \simeq d + \frac{\sqrt{3}}{2}\left(\epsilon_{2rsx} - \epsilon_{(2r+1)(s-1)x}\right) + \frac{1}{2}\epsilon_{(2r+1)(s-1)y}$$

$$D_{2rs,(2r+1)s} \simeq d + \frac{\sqrt{3}}{2}\left(\epsilon_{2rsx} - \epsilon_{(2r+1)sx}\right) - \frac{1}{2}\left(\epsilon_{2rsy} - \epsilon_{(2r+1)sy}\right)$$

Thus,

$D^{(0)}_{ij,k\ell} = d = 8{,}000$ for all

$a = \frac{\sqrt{3}}{2}$ or 0

$b = \pm\frac{1}{2}$

And the resulting set of equations (for $w_i = 1$ for all weights) is

$$3\frac{\sqrt{3}}{2}\epsilon_{11x} - \frac{\sqrt{3}}{2}\epsilon_{21x} - \sqrt{3}\,\epsilon_{22x} - \frac{1}{2}\epsilon_{11y} - \frac{1}{2}\epsilon_{21y} + \epsilon_{22y}$$

$$= \left[\left(D^{(1)}_{11,22} - d\right) + \left(D^{(1)}_{11,22} - d\right) + \left(D^{(2)}_{11,22} - d\right)\right] \qquad (1)$$

$$2\sqrt{3}\,\epsilon_{1sx} - \sqrt{3}\,\epsilon_{2sx} - \sqrt{3}\,\epsilon_{2(s+1)x} - \epsilon_{2sy} + \epsilon_{2(s+1)y}$$

$$= \left[\left(D^{(1)}_{1s,2s} - d\right) + \left(D^{(2)}_{1s,2s} - d\right) + \left(D^{(1)}_{1s,2(s+1)}\right) + \left(D^{(2)}_{1s,2(s+1)} - d\right)\right]$$

$(s = 2,\ldots,6)$ (2 through 6)

$$3\frac{\sqrt{3}}{2}\epsilon_{17x} - \sqrt{3}\,\epsilon_{27x} - \frac{\sqrt{3}}{2}\epsilon_{28x} + \frac{1}{2}\epsilon_{17y} + \epsilon_{27y} + \frac{1}{2}\epsilon_{28}$$

$$= \left[\left(D^{(1)}_{17,27} - d\right) + \left(D^{(2)}_{17,27} - d\right) + \left(D^{(1)}_{17,28} - d\right)\right] \qquad (7)$$

$$-\frac{\sqrt{3}}{2}\epsilon_{(2r-1)x} + \sqrt{3}\,\epsilon_{2r1x} - \frac{\sqrt{3}}{2}\epsilon_{(2r+1)1x} - \frac{1}{2}\epsilon_{(2r-1)1y} + \epsilon_{(2r+1)1y}$$

$$= \left[\left(D^{(1)}_{(2r)1,(2r+1)1}\right) - \left(D^{(2)}_{(2r-1)1,(2r)1} - d\right)\right]$$

$(r = 1,2)$ (8, 21)

$$-\sqrt{3}\,\epsilon_{(2r-1)(s-1)x} - \sqrt{3}\,\epsilon_{(2r-1)sx} + 4\sqrt{3}\,\epsilon_{2rsx} - \sqrt{3}\,\epsilon_{(2r+1)sx} + \epsilon_{(2r-1)(s-1)y} - \epsilon_{(2r-1)sy}$$

$$- \epsilon_{(2r+1)(s-1)y} + \epsilon_{(2r+1)sy}$$

$$= \left[\left(D^{(1)}_{2rs,(2r+1)(s-1)} - d\right) + \left(D^{(2)}_{2rs,(2r+1)(s-1)} - d\right) + \left(D^{(1)}_{2rs,(2r+1)s} - d\right)\right.$$

$$+ \left(D^{(2)}_{2rs,(2r+1)s} - d\right) - \left(D^{(1)}_{(2r-1)s,2rs} - d\right) - \left(D^{(2)}_{(2r-1)s,2rs} - d\right)$$

$$\left. - \left(D^{(1)}_{(2r-1)(s-1)2rs} - d\right) - \left(D^{(2)}_{(2r-1)(s-1),2rs} - d\right)\right]$$

$(r = 1,2;\ s = 2,\ldots,7)$ (9 through 14 and 22 through 27)

$$-\frac{\sqrt{3}}{2}\,\epsilon_{(2r-1)7x}+\sqrt{3}\,\epsilon_{2r8x}-\frac{\sqrt{3}}{2}\,\epsilon_{(2r+1)7x}+\frac{1}{2}\,\epsilon_{(2r-1)7y}-\frac{1}{2}\,\epsilon_{(2r+1)7y}$$

$$=\left[\left(D^{(1)}_{2r8,(2r+1)7}-d\right)-\left(D^{(1)}_{(2r-1)7,2r8}-d\right)\right]$$

$(r = 1,2)$ (15, 28)

$$-\frac{\sqrt{3}}{2}\,\epsilon_{21x}-\sqrt{3}\,\epsilon_{22x}+3\sqrt{3}\,\epsilon_{31x}-\frac{\sqrt{3}}{2}\,\epsilon_{41x}-\sqrt{3}\,\epsilon_{42x}+\frac{1}{2}\epsilon_{21y}-\epsilon_{22y}-\frac{1}{2}\epsilon_{41y}+\epsilon_{42y}$$

$$=\left[\left(D^{(1)}_{31,41}-d\right)+\left(D^{(1)}_{31,42}-d\right)+\left(D^{(2)}_{31,42}-d\right)-\left(D^{(1)}_{22,31}-d\right)-\left(D^{(2)}_{22,31}-d\right)\right.$$

$$\left.-\left(D^{(1)}_{21,31}-d\right)\right] \tag{16}$$

$$-\sqrt{3}\,\epsilon_{2sx}-\sqrt{3}\,\epsilon_{2(s+1)x}+4\sqrt{3}\,\epsilon_{3sx}-\sqrt{3}\,\epsilon_{4sx}-\sqrt{3}\,\epsilon_{4(s+1)x}+\epsilon_{2sy}-\epsilon_{2(s+1)y}-\epsilon_{4sy}+\epsilon_{4(s+1)y}$$

$$=\left[\left(D^{(1)}_{3s,4s}-d\right)+\left(D^{(2)}_{3s,4s}-d\right)+\left(D^{(1)}_{3s,4(s+1)}-d\right)+\left(D^{(2)}_{3s,4(s+1)}-d\right)-\left(D^{(1)}_{2s,3s}-d\right)\right.$$

$$\left.-\left(D^{(2)}_{2s,3s}-d\right)-\left(D^{(1)}_{2(s+1),3s}-d\right)-\left(D^{(2)}_{2(s+1),3s}-d\right)\right]$$

$(s = 2,3,5,6)$ (17 through 20)

$$-\frac{\sqrt{3}}{2}\,\epsilon_{41x}-\sqrt{3}\,\epsilon_{42x}+3\frac{\sqrt{3}}{2}\,\epsilon_{51x}+\frac{1}{2}\epsilon_{41y}-\epsilon_{42y}+\frac{1}{2}\epsilon_{51y}$$

$$=\left[\left(D^{(1)}_{42,51}-d\right)+\left(D^{(2)}_{42,51}-d\right)+\left(D^{(1)}_{41,51}-d\right)\right] \tag{29}$$

$$-\sqrt{3}\,\epsilon_{4sx}-\sqrt{3}\,\epsilon_{4(s+1)x}+2\sqrt{3}\,\epsilon_{5sx}+\epsilon_{4sy}-\epsilon_{4(s+1)y}$$

$$=\left[-\left(D^{(1)}_{4(s+1),5s}-d\right)-\left(D^{(2)}_{4(s+1),5s}-d\right)+\left(D^{(1)}_{4s,5s}-d\right)+\left(D^{(2)}_{4s,5s}-d\right)\right]$$

$(s = 2,\ldots,6)$ (30 through 34)

$$-\sqrt{3}\,\epsilon_{47x}-\frac{\sqrt{3}}{2}\,\epsilon_{48x}+3\frac{\sqrt{3}}{2}\,\epsilon_{57x}+\epsilon_{47y}-\frac{1}{2}\epsilon_{48y}-\frac{1}{2}\epsilon_{57y}$$

$$=\left[-\left(D^{(1)}_{48,57}-d\right)-\left(D^{(1)}_{47,57}-d\right)-\left(D^{(2)}_{47,57}-d\right)\right] \tag{35}$$

$$-\sqrt{3}\,\epsilon_{11x} - \frac{\sqrt{3}}{4}\epsilon_{21x} + \frac{\sqrt{3}}{2}\epsilon_{22x} + \frac{7}{4}\epsilon_{11y} - \epsilon_{12y} - \epsilon_{21y} - \frac{1}{2}\epsilon_{22y}$$

$$= \left\{-\left(D^{(1)}_{11,12} - d\right) + \frac{1}{2}\left[\left(D^{(1)}_{11,21} - d\right) - \left(D^{(1)}_{11,22} - d\right) - \left(D^{(2)}_{11,22}\right)\right]\right\} \tag{36}$$

$$-\frac{\sqrt{3}}{2}\epsilon_{2sx} + \frac{\sqrt{3}}{2}\epsilon_{2(s+1)x} - \epsilon_{1(s-1)y} + 3\,\epsilon_{1sy} - \epsilon_{1(s+1)y} - \frac{1}{2}\epsilon_{2sy} - \frac{1}{2}\epsilon_{2(s+1)y}$$

$$= \left\{\left(D^{(1)}_{1(s-1),1s} - d\right) - \left(D^{(1)}_{1s,1(s+1)} - d\right) - \frac{1}{2}\left[\left(D^{(1)}_{1s,2(s+1)} - d\right) + \left(D^{(2)}_{1s,2(s+1)} - d\right) - \left(D^{(1)}_{1s,2s} - d\right)\right.\right.$$

$$\left.\left. - \left(D^{(2)}_{1s,2s} - d\right)\right]\right\}$$

$(s = 2,\ldots,6)$ (37 through 41)

$$\frac{\sqrt{3}}{4}\epsilon_{17x}\ \frac{\sqrt{3}}{4}\epsilon_{27x} + \frac{\sqrt{3}}{2}\epsilon_{28x} - \epsilon_{16y} + \frac{7}{4}\epsilon_{17y} - \frac{1}{2}\epsilon_{27y} - \frac{1}{4}\epsilon_{28y}$$

$$= \left\{-\left(D^{(1)}_{16,17} - d\right) + \frac{1}{2}\left[\left(D^{(1)}_{17,28} - d\right) - \left(D^{(1)}_{17,27} - d\right) - \left(D^{(2)}_{17,27} - d\right)\right]\right\} \tag{42}$$

$$-\frac{\sqrt{3}}{4}\epsilon_{(2r-1)1x} + \frac{\sqrt{3}}{4}\epsilon_{(2r+1)1x} - \frac{1}{4}\epsilon_{(2r-1)1y} + \frac{5}{2}\epsilon_{(2r)1y} - 2\,\epsilon_{2r2y} - \frac{1}{4}\epsilon_{(2r+1)1y}$$

$$= \left\{-\left(D^{(1)}_{(2r)1,(2r)2} - d\right) - \left(D^{(2)}_{(2r)1,2r2} - d\right) + \frac{1}{2}\left[\left(D^{(1)}_{(2r-1)1,(2r1)} - d\right) + \left(D^{(1)}_{(2r)1,(2r+1)} - d\right)\right]\right\}$$

$(r = 1,2)$ (43, 57)

$$\frac{\sqrt{3}}{2}\epsilon_{(2r-1)(s-1)x} - \frac{\sqrt{3}}{2}\epsilon_{(2r-1)sx} - \frac{\sqrt{3}}{2}\epsilon_{(2r+1)(s-1)x} + \frac{\sqrt{3}}{2}\epsilon_{(2r+1)sx} - \frac{1}{2}\epsilon_{(2r-1)(s-1)y}$$

$$-\frac{1}{2}\epsilon_{(2r-1)sy} - 2\,\epsilon_{2r(s-1)y} + 6\,\epsilon_{(2r)(2+1)y} - \frac{1}{2}\epsilon_{(2r+1)(s-1)y} - \frac{1}{2}\epsilon_{(2r+1)sy}$$

$$= \left\{\left(D^{(1)}_{2r(s-1),2rs} - d\right) + \left(D^{(2)}_{2r(d-1),2rs} - d\right) - \left(D^{(1)}_{2rs,2r(s+1)} - d\right) - \left(D^{(2)}_{2rs,2r(s+1)} - d\right)\right.$$

$$-\frac{1}{2}\left[\left(D^{(1)}_{(2r-1)s,2rs} - d\right) + \left(D^{(2)}_{(2r-1)s,2rs} - d\right) + \left(D^{(1)}_{2rs,(2r+1)s} - d\right) + \left(D^{(2)}_{2rs,(2r+1)s} - d\right)\right.$$

$$- \left(D^{(1)}_{(2r-1)(s-1),2rs} - d\right) - \left(D^{(2)}_{(2r-1)(s-1),2rs} - d\right) - \left(D^{(1)}_{2rs,(2r+1)(s-1)} - d\right)$$

$$\left.\left. - \left(D^{(2)}_{2rs,(2r+1)(s-1)} - d\right)\right]\right\}$$

$(r = 1,2;\quad s = 2,\ldots,7)$ (44 through 49 and 58 through 63)

$$\frac{\sqrt{3}}{4}\,\epsilon_{(2r-1)7x}-\frac{\sqrt{3}}{4}\,\epsilon_{(2r+1)7x}-\frac{1}{4}\,\epsilon_{(2r-1)7y}-2\,\epsilon_{2r7y}+\frac{5}{2}\,\epsilon_{2(4+1)y}-\frac{1}{4}\,\epsilon_{(2r+1)7y}$$

$$=\left\{\left(D^{(1)}_{2r7,2r8}-d\right)+\left(D^{(2)}_{2r7,2r8}-d\right)+\frac{1}{2}\left[\left(D^{(1)}_{(2r-1)7,2r8}-d\right)+\left(D^{(1)}_{2r8,(2r+1)7}-d\right)\right]\right\}$$

$(r = 1,2)$ (50, 64)

$$\frac{\sqrt{3}}{4}\,\epsilon_{21x}-\frac{\sqrt{3}}{4}\,\epsilon_{22X}-\frac{\sqrt{3}}{4}\,\epsilon_{41x}+\frac{\sqrt{3}}{2}\,\epsilon_{42x}-\frac{1}{4}\,\epsilon_{21y}-\frac{1}{2}\,\epsilon_{22y}+\frac{7}{2}\,\epsilon_{31y}-2\,\epsilon_{32y}-\frac{1}{4}\,\epsilon_{41y}-\frac{1}{2}\,\epsilon_{42y}$$

$$=\left[-\left(D^{(1)}_{31,32}-d\right)-\left(D^{(2)}_{31,32}-d\right)-\left(D^{(1)}_{22,31}-d\right)+\left(D^{(2)}_{22,31}-d\right)+\left(D^{(1)}_{31,42}-d\right)+\left(D^{(2)}_{31,42}-d\right)\right.$$

$$\left.-\left(D^{(1)}_{31,41}-d\right)-\left(D^{(1)}_{21,31}-d\right)\right] \qquad (51)$$

$$\frac{\sqrt{3}}{2}\,\epsilon_{2sx}-\frac{\sqrt{3}}{2}\,\epsilon_{2(s+1)x}-\frac{\sqrt{3}}{2}\,\epsilon_{4sx}+\frac{\sqrt{3}}{2}\,\epsilon_{4(s+1)x}-\frac{1}{2}\,\epsilon_{2sy}-\frac{1}{2}\,\epsilon_{2(s+1)y}-2\epsilon_{3(s-1)y}+6\epsilon_{3sy}$$

$$-2\epsilon_{3(s+1)y}-\frac{1}{2}\,\epsilon_{4(s+1)y}$$

$$=\left\{\left(D^{(1)}_{3(s-1),3s}-d\right)+\left(D^{(2)}_{3(s-1),3s}-d\right)-\left(D^{(1)}_{3s,3(s+1)}-d\right)-\left(D^{(2)}_{3s,3(s-1)}-d\right)-\frac{1}{2}\left[\left(D^{(1)}_{2(s+1),3s}-d\right)\right.\right.$$

$$+\left(D^{(2)}_{2(s+1),3s}-d\right)+\left(D^{(1)}_{3s,4(s+1)}-d\right)+\left(D^{(2)}_{3s,4(s+1)}-d\right)-\left(D^{(1)}_{2s,3s}-d\right)-\left(D^{(2)}_{2s,3s}-d\right)$$

$$\left.\left.-\left(D^{(1)}_{3s,4s}-d\right)-\left(D^{(2)}_{3s,4s}-d\right)\right]\right\}$$

$(s = 2,3,5,6)$ (52 through 55)

$$\frac{\sqrt{3}}{2}\,\epsilon_{27x}-\frac{\sqrt{3}}{4}\,\epsilon_{28x}-\frac{\sqrt{3}}{2}\,\epsilon_{47x}+\frac{\sqrt{3}}{4}\,\epsilon_{48x}-\frac{1}{2}\,\epsilon_{27y}-\frac{1}{4}\,\epsilon_{28y}-2\,\epsilon_{36y}+\frac{7}{2}\,\epsilon_{37y}-\frac{1}{2}\,\epsilon_{47y}-\frac{1}{4}\,\epsilon_{48y}$$

$$=\left\{\left(D^{(1)}_{36,37}-d\right)+\left(D^{(2)}_{36,37}-d\right)-\frac{1}{2}\left[\left(D^{(1)}_{37,48}-d\right)+\left(D^{(2)}_{28,37}-d\right)-\left(D^{(1)}_{27,37}-d\right)-\left(D^{(2)}_{27,37}-d\right)\right.\right.$$

$$\left.\left.-\left(D^{(1)}_{37,47}-d\right)-\left(D^{(2)}_{37,47}-d\right)\right]\right\} \qquad (56)$$

$$\frac{\sqrt{3}}{4}\,\epsilon_{41x}-\frac{\sqrt{3}}{2}\,\epsilon_{42x}+\frac{\sqrt{3}}{4}\,\epsilon_{51x}-\frac{1}{4}\,\epsilon_{41y}-\frac{1}{2}\,\epsilon_{42y}+\frac{7}{4}\,\epsilon_{51y}-2\,\epsilon_{52y}$$

$$=\left\{-\left(D^{(1)}_{51,52}-d\right)-\frac{1}{2}\left[\left(D^{(1)}_{42,51}-d\right)+\left(D^{(2)}_{42,51}-d\right)-\left(D^{(1)}_{41,51}-d\right)\right]\right\} \qquad (65)$$

$$\frac{\sqrt{3}}{2}\epsilon_{45x} - \frac{\sqrt{3}}{2}\epsilon_{4(s+1)x} - \frac{1}{2}\epsilon_{4sy} - \frac{1}{2}\epsilon_{4(s+1)y} - \epsilon_{5(s-1)y} + 3\epsilon_{5sy} - \epsilon_{5(s+1)y}$$

$$= \left\{\left(D^{(1)}_{5(s-1),5s} - d\right) - \left(D^{(1)}_{5s,5(s+1)} - d\right) - \frac{1}{2}\left[\left(D^{(1)}_{4(s+1),5s} - d\right) + \left(D^{(2)}_{4(s+1),5s} - d\right)\right.\right.$$

$$\left.\left. - \left(D^{(1)}_{4s,5s} - d\right) - \left(D^{(1)}_{4s,5s} - d\right)\right]\right\}$$

$(s = 2,\ldots,6)$ (66 through 70)

$$\frac{\sqrt{3}}{2}\epsilon_{47x} - \frac{\sqrt{3}}{4}\epsilon_{48x} - \frac{\sqrt{3}}{4}\epsilon_{57x} - \frac{1}{2}\epsilon_{47y} - \frac{1}{4}\epsilon_{48y} - \epsilon_{56y} + \frac{7}{4}\epsilon_{57y}$$

$$= \left[\left(D^{(1)}_{56,57} - d\right) - \frac{1}{2}\left(D^{(1)}_{48,57} - d\right) - \left(D^{(1)}_{47,57} - d\right) - \left(D^{(2)}_{47,57} - d\right)\right] \quad (71)$$

Thus,

$$X^{(1)} = X^{(0)} + \epsilon_{ijx}$$

$$Y^{(1)} = Y^{(0)} + \epsilon_{ijy}$$

and

$$D^{(1)}_{ij,k\ell} = \sqrt{\left(X^{(1)}_{ij} - X^{(1)}_{k\ell}\right)^2 + \left(Y^{(1)}_{ij} - Y^{(1)}_{k\ell}\right)^2}$$

and the iteration is continued until $\left| X^{(N)}_{ij} - X^{(N+1)}_{ij} \right| < \epsilon_1$. $\left| Y^{(N)}_{ij} - Y^{(N+1)}_{ij} \right| < \epsilon_2$ for all i,j.

To further simplify the program it was also assumed that $a^{(N-1)}_{ij,k\ell}$ = constant = $\pm\frac{\sqrt{3}}{2}$ or 0, and $b^{(N-1)}_{ij,k\ell}$ = constant = $\pm\frac{1}{2}$ or 0. The quality of the results obtained imply that this assumption is valid, and it does not seem necessary or worthwhile to include the non-constant portion of the coefficients in the calculation.

The system may be written more concisely in matrix notation

$$A\Delta\hat{X} = \left(\hat{b} - \hat{b}(N)\right)$$

where

A = the matrix of coefficients

$\Delta\hat{X}$ = the vector of displacement from the previous best estimate of position

$\hat{b}$ = a vector composed of linear combinations of the interhydrophone distances $(\hat{d})$

$\hat{b}(N)$|= the vector of corresponding N*th* estimates

A further consequence of the linear least-squares formulation of the problem is an estimate of the propagated error in the coordinates $\hat{X} = \hat{X}(N-1) + A^{-1}\left[\hat{b} - \hat{b}(N-1)\right]$. Thus it can be shown (see appendix) that $\sigma_{\hat{x}} = A^{-1}\,\sigma_{\hat{b}}\,\left(A^{-1}\right)^T$. Further, $\hat{b} = C\hat{d}$, so that, finally, $\sigma_{\hat{x}} = A^{-1}C\,\sigma_{\hat{d}}\,\left(A^{-1}C\right)^T$, and for an estimate of $\sigma_{\hat{d}} = \mathrm{DIAG}\left(\sigma^2_{d1}, \sigma^2_{d2}, \ldots, \sigma^2_{d156}\right)$, taking the diagonal elements of $\sigma_{\hat{x}}$ will give an estimate of propagated errors in each of the hydrophone coordinates resulting from the uncertainty in the interhydrophone distance measurements.

SUMMARY OF RESULTS

Although only one Vanderkulk Survey has been run at the Pacific Missile Range Facility, Hawaiian Area Underwater Range, four sets of interhydrophone distances were used as input, resulting in four sets of hydrophone positions. The first set is the original unedited results obtained by ITT (International Telephone and Telegraph Corporation). The second set is the ITT data with certain distances edited. The ITT results with this second set of interhydrophone distances are currently in use at this range. The third set was obtained at the Pacific Missile Range by the Mathematical Analysis Branch (Code 3433), the interhydrophone distances were recomputed with a greater amount of data per triad, and the resulting hydrophone positions were recalculated. The fourth set of input resulted from an editing of the Pacific Missile Range data.

With these four sets as input, four least-squares solutions were obtained. Table 1 is a tabular listing of the difference between the results of the least-squares run 4 (taken as the standard) and the other runs for corresponding coordinates. The difference between the present set of coordinates and the proposed set is presented graphically in figure 2. As would be expected, the results from the least-squares formulation are much more consistent.

Table 2 lists the estimate of propagated error in each coordinate caused by the uncertainties in the interhydrophone distance measurements. The average value is about 5 feet.

As a further consistency check, the interhydrophone distances were computed with the use of the calculated hydrophone positions. Here, the results are quite revealing; the difference between the interhydrophone distances computed and the weighted average of the measured distances

$\bar{d} = \left[\sigma_2 / \left(\sigma_1 + \sigma_2\right)\right] d_1 + \left[\sigma_1 / \left(\sigma_1 + \sigma_2\right)\right] d_2$ is tabulated below:

Difference (Feet)	ITT(1)	LS(1)	ITT(2)	LS(2)	PMR	LS(3)
<2	60	68	66	83	63	84
2 to 5	9	19	14	5	17	4
5 to 10	6	1	8		8	.
10 to 15	7					
15 to 20	3					
Over 20	3					

NOTE: LS = least squares.

Table 1. Difference in Coordinate Position From Those of ST

Hydro-phone	ITT(1)-ST	ITT(2)-ST	PMR-ST	SLSS(1)-ST	SLSS(2)-ST	SLSS(3)-ST
11	-30.65	24.84	5.86	-4.39	17.20	-1.42
	32.71	-0.32	3.00	7.26	-0.18	-0.30
12	-15.85	20.40	1.62	-2.27	11.38	-1.26
	28.00	-4.58	3.80	2.04	-5.32	-0.44
13	-8.44	9.02	-2.82	-7.29	0.39	-2.00
	26.26	-6.59	3.99	1.66	-4.81	-0.89
14	8.64	7.97	7.49	-4.32	5.47	-0.48
	27.84	-4.45	5.16	1.31	-3.25	-0.85
15	18.80	11.18	5.26	-3.70	7.46	-0.96
	29.19	-3.02	4.75	2.98	-4.22	-0.80
16	25.31	6.55	6.40	-2.23	3.90	-1.46
	29.44	-2.56	4.84	4.12	-4.01	-0.69
17	12.75	-5.34	-2.88	-5.95	-5.54	-1.29
	34.66	2.66	5.83	8.83	0.47	-0.45
21	-34.80	29.11	11.63	-5.60	19.06	-1.62
	17.00	0.03	12.54	-1.10	-2.48	-0.36
22	-21.32	24.38	3.82	-2.60	15.32	-1.35
	15.41	-1.30	5.16	3.22	1.98	-0.29
23	-13.24	13.53	-2.88	-0.51	9.82	-1.26
	14.55	-2.41	3.38	4.39	1.45	-0.15
24	-0.92	7.35	3.74	-2.69	3.90	-0.10
	11.68	-4.77	0.48	2.36	-3.20	-0.11
25	8.80	10.60	6.27	-3.79	9.21	-0.69
	9.94	-6.22	5.17	0.15	-5.50	-0.41
26	16.47	6.67	8.54	-2.43	6.29	-1.22
	10.44	-5.75	5.04	1.56	-2.80	-0.39
27	9.43	0.99	5.31	-1.39	1.81	-1.56
	10.77	-5.41	5.00	1.24	-2.77	-0.67
28	2.62	-6.62	1.53	-1.31	-3.74	-0.82
	11.01	-5.16	3.26	1.57	-2.23	-0.79
31	-34.04	28.87	7.95	-4.22	16.86	-1.65
	-5.05	4.74	7.71	1.23	6.92	-0.61
32	-20.57	22.17	1.83	-1.26	14.05	-1.23
	-1.33	8.58	9.68	2.47	9.13	-0.50
33	-5.28	13.27	1.17	-0.10	10.03	-0.85
	1.60	0.60	-3.25	2.30	1.01	-0.46
34	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00
35	12.81	12.70	4.81	-4.11	6.90	-1.26
	1.39	1.40	0.48	0.32	0.78	-0.02
36	2.66	3.87	0.96	-1.92	3.96	-1.13
	1.30	1.32	-0.42	-0.06	0.78	-0.03
37	0.00	0.00	0.00	0.00	0.00	0.00
	2.14	2.15	-1.40	0.57	1.50	0.16
41	-49.39	16.03	4.94	-19.25	2.96	-1.58
	-2.24	12.38	0.24	3.06	11.66	-0.57
42	-21.94	24.80	3.93	-5.08	13.21	-1.15
	-4.24	11.57	4.01	-1.21	7.80	-0.62
43	-8.86	20.36	1.72	-0.51	14.07	-1.01
	-6.78	8.98	-0.59	-2.72	5.49	-0.80
44	-2.95	6.62	-0.46	-1.92	5.67	-0.83
	-6.49	9.29	-1.13	-3.04	5.75	-1.01
45	2.89	12.50	0.43	-2.22	13.02	-2.10
	-8.27	7.51	-1.75	-2.05	6.83	-1.10
46	-6.01	3.64	-1.18	-3.81	4.74	-1.27
	-9.51	6.27	-2.23	-1.12	5.68	-0.70
47	-6.99	2.69	-0.09	-0.89	2.09	-0.60
	-9.62	6.16	-2.07	-1.29	5.03	-0.70
48	-13.75	-4.03	-3.45	0.43	-1.62	0.24
	-9.58	6.20	-2.24	-1.68	5.11	-0.73
51	-25.43	21.02	-3.36	-11.65	8.47	-1.34
	-6.84	8.42	-0.63	-8.74	3.35	-0.92
52	-7.18	21.99	2.91	-2.36	14.20	-1.02
	-6.08	9.57	0.03	-7.04	4.80	-0.87
53	-0.57	8.98	-0.12	-1.63	9.18	-0.86
	-3.85	11.90	-0.69	-2.58	8.90	-0.87
54	-3.57	6.02	-0.07	-6.51	4.75	-1.40
	-4.87	10.88	-1.57	-0.06	8.16	-0.66
55	-0.61	9.01	0.14	-3.44	9.62	-1.84
	-2.92	12.82	-1.43	0.20	11.16	-1.17
56	-6.53	3.13	-0.96	-2.97	3.56	-1.01
	-4.42	11.32	-1.83	-1.13	10.22	-1.29
57	-9.55	0.15	-1.25	0.34	0.73	-0.20
	-4.82	10.93	-0.67	-2.50	8.73	-1.36

Note: SLSS(1) = Least-squares results from same input as used for ITT(1).
SLSS(2) = Least-squares results from same input as used for ITT(2).
SLSS(3) = Least-squares results from same input as used for PMR.
ST = Least-squares results from edited PMR input.

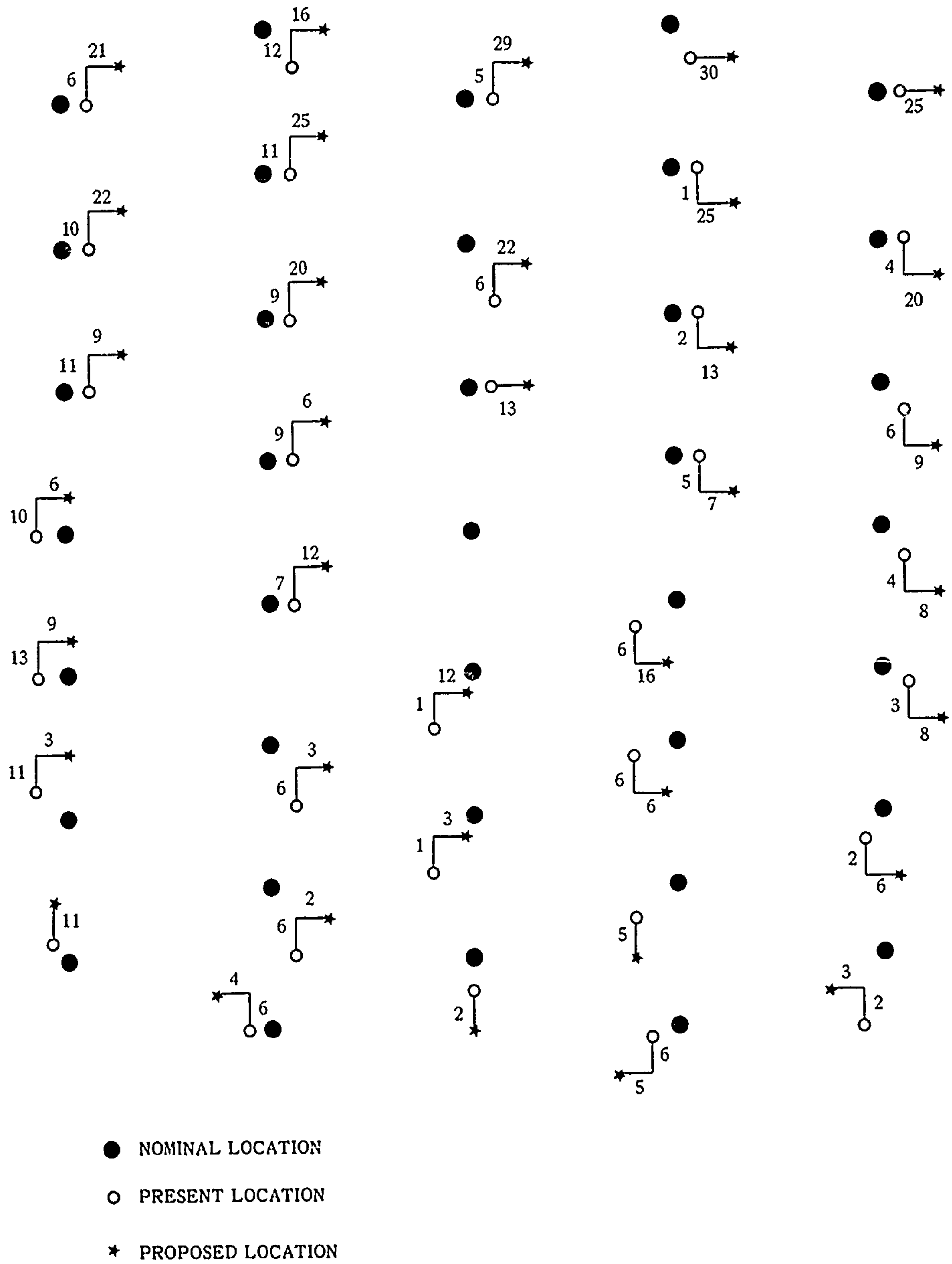

Figure 2. Comparative Hydrophone Coordinates.

Table 2. Estimate of Propagated Error

Hydro-phone	1 σ Error (Feet)	Hydro-phone	1 σ Error (Feet)
11Y	7.78	33X	3.64
11X	9.73	35Y	2.88
12Y	5.38	35X	2.88
12X	6.46	36Y	3.06
13Y	5.00	36X	3.05
13X	4.73	37Y	3.87
14Y	4.67	41Y	4.68
14X	2.70	41X	11.63
15Y	4.30	42Y	4.09
15X	3.27	42X	7.51
16Y	4.54	43Y	3.69
16X	4.16	43X	4.95
17Y	4.79	44Y	3.31
17X	5.14	44X	2.80
21Y	5.91	45Y	3.27
21X	12.47	45X	3.28
22Y	3.73	46Y	3.31
22X	7.46	46X	3.22
23Y	3.26	47Y	3.40
23X	4.93	47X	2.16
24Y	3.03	48Y	3.76
24X	2.51	48X	3.08
25Y	2.85	51Y	6.96
25X	2.30	51X	9.62
26Y	3.13	52Y	5.87
26X	3.35	52X	6.40
27Y	3.66	53Y	5.58
27X	4.37	53X	4.19
28Y	4.40	54Y	5.18
28X	4.66	54X	3.41
31Y	3.71	55Y	4.41
31X	3.95	55X	3.67
32Y	2.83	56Y	4.31
32X	6.06	56X	3.31
33Y	2.45	57Y	5.13
		57X	2.75

CONCLUSIONS

A study of table 1 and figure 2 reveals two very important phenomena. The first is that the least-squares solution obtained with the unedited ITT data actually agrees with the assumed standard much better than the least-squares solution obtained from the edited ITT data. The second is that, relative to the assumed standard, the range appears to be slightly rotated, or the fourth and fifth rows appear to be biased in the Y-direction; the fourth row by about 8 feet and the fifth row by about 10 feet.

As was previously stated, these results were obtained with the use of simplifying assumptions, and future studies are planned to show the effect of different weighting schemes, and possibly also of the introduction of the complete coefficient matrix (including the functional dependence).

The present program was developed to test the feasibility of the technique, and this has been amply verified. The convergence of the solution is such that after five iterations, the hydrophone coordinates are consistent to within a maximum of 0.1 foot.

A further benefit is that the entire run (five iterations) is completed in less than 2 minutes of computer time rather than the days or weeks required by the present method that includes hand calculations. Also, as previously stated, there is an automatic error estimate.

A further advantage of this technique is that, as future surveys are performed, these data can be combined with previous survey information to yield an updated best estimate that would include all available information.

Obviously, this technique can be applied to any other underwater range. The only difference would be in the formulation of the initial estimates.

This report has been devoted to finding only the X-Y grid coordinates of the hydrophones. Since the Vanderkulk survey measures the depth of each hydrophone directly, all that is required for the Z-coordinates is a weighted average of these measurements.

APPENDIX

DERIVATION OF PROPAGATED ERROR ESTIMATE

At any stage of the iteration, $\hat{X}(N) - \hat{X}(N-1) = A^{-1}\left(\hat{b} - \hat{b}(N-1)\right)$, or, in the component form, $X_j(N) - X_j(N-1) = \sum_k A_{jk}^{-1}\left(b_k - b_k(N-1)\right)$.

Thus,

$$\frac{\partial x_j(N)}{\partial b_m} = \frac{\partial x_j(N-1)}{\partial b_m} + A_{jk}^{-1} - \sum_k A_{jk}^{-1} \frac{\partial b_k(N-1)}{\partial b_m}$$

Further,

$$\sum_k A_{jk}^{-1} \frac{\partial b_k(N-1)}{\partial b_m} = \left(\sum_k A_{ij}^{-1} \sum_\ell \frac{\partial b_k(N-1)}{\partial x_\ell} \frac{\partial x_\ell}{\partial b_m}\right)$$

Now, if $b_k(x)$ is expanded in a Taylor series,

$$b_k(x) = b_k(N-1) + \left(\sum_\ell \frac{\partial b_k}{\partial x_\ell}\right)\Delta x + O\left((\Delta x)^2\right)$$

or

$$A_{k\ell} \simeq \frac{\partial b_k}{\partial x_\ell}$$

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Thus, since

$$\sum_k A_{jk}^{-1} A_{k\ell} = \delta_{j\ell}$$

$$\sum_k A_{jk}^{-1}\left(\sum_\ell \frac{\partial b_k}{\partial x}\,\frac{\partial x_\ell}{\partial b_m}\right) = \sum_\ell\left(\sum_k A_{jk}^{-1}\,\frac{\partial b_k}{\partial x_\ell}\,\frac{\partial x_\ell}{\partial b_m}\right) \simeq \sum_\ell\left(\sum_k A_{jk}^{-1} A_{k\ell}\,\frac{\partial x_\ell}{\partial b_m}\right) \simeq \frac{\partial x_j}{\partial b_m}$$

or

$$\frac{\partial x_j}{\partial b_m} \simeq \frac{\partial x_j(N-1)}{\partial b_m} + A_{jkm}^{-1} - \frac{\partial x_j(N-1)}{\partial b_m} = A_{jkm}^{-1}$$

Therefore

$$\sigma_{\hat{x}} \simeq A^{-1}\,\sigma_{\hat{b}}\left(A^{-1}\right)^T$$

NOTE: The preceding derivation holds only when A is assumed to be a matrix of constants; when the full coefficient is used a slightly different formula arises. The complete formulation appears in a Pacific Missile Range unpublished Working Note 3433-21, Propagation of Error Associated with an Iterative Least-Squares Solution, by S. Berman, dated December 1967.

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13. ABSTRACT

The purpose of this report is to develop the mathematics for a least-squares method to simultaneously locate an array of hydrophones through the use of the interhydrophone distances computed by the method derived by W. Vanderkulk.

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